Accurate and Interpretable Radar Quantitative Precipitation Estimation with Symbolic Regression

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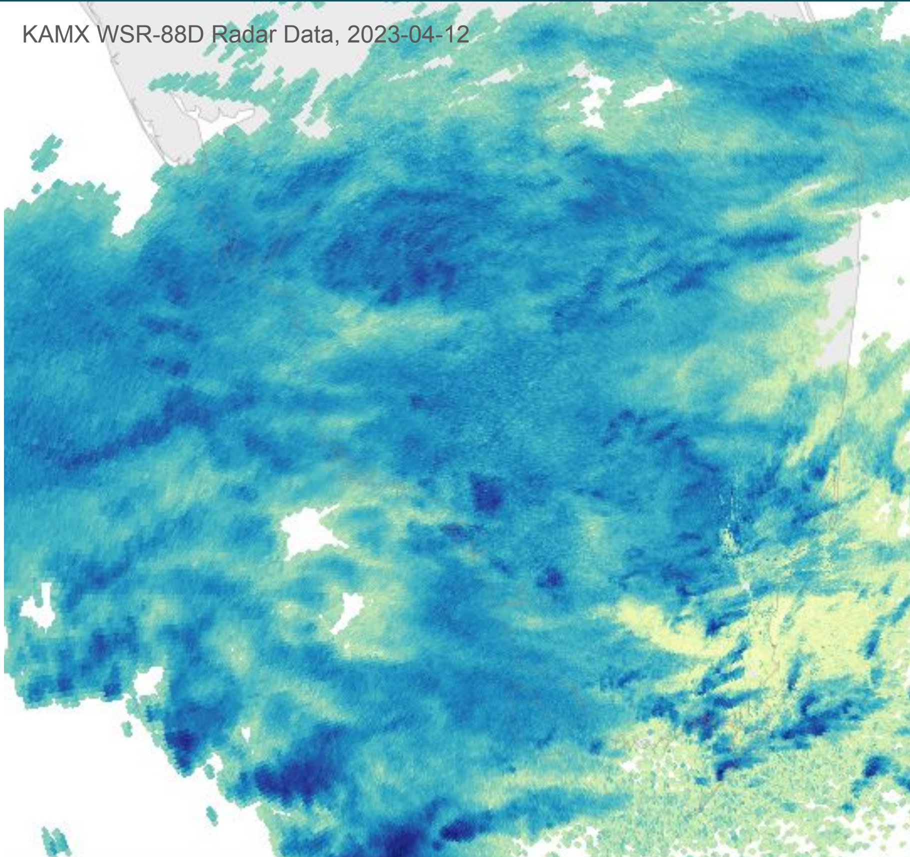


1. Introduction

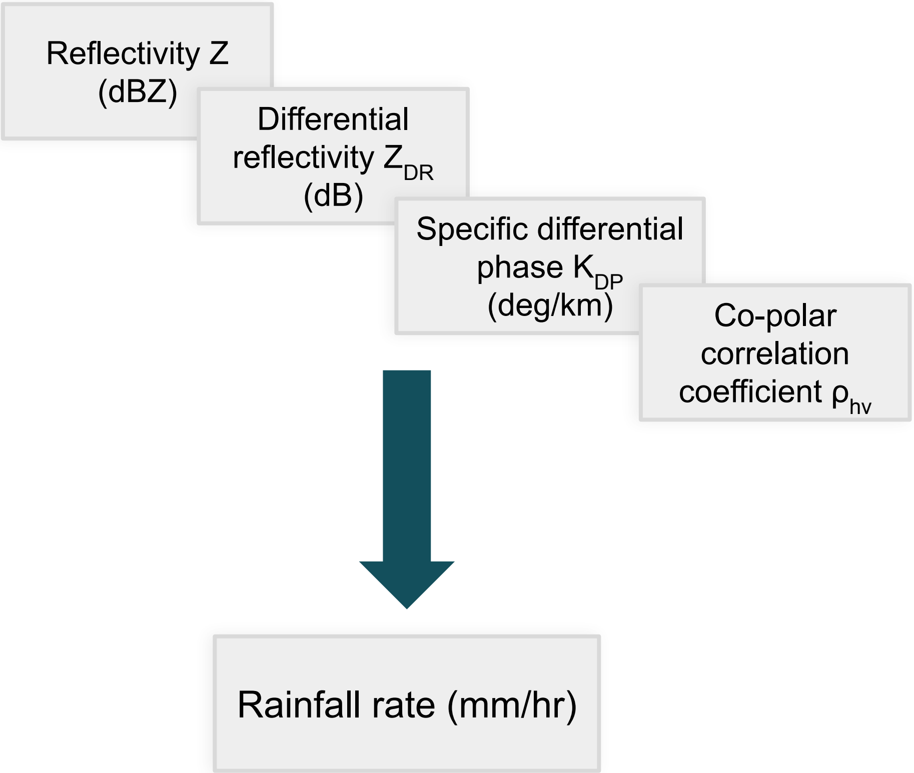
- Accurate estimation of precipitation is crucial for a variety of applications such as extreme weather condition forecasting, flash flood management, and ongoing climate research [9].
- However, quantitative precipitation estimation (QPE) is limited through the following methods:
    - Collecting rainfall from rain gauges has limited spatial coverage.
    - Estimating rainfall from single-polarimetric radar data may fail to account for different precipitation types and intensities [2].
- Our research focuses on improving QPE using dual-polarimetric radar data with symbolic regression.
    - Symbolic regression provides a unique approach by providing interpretable and accurate equations learned from data [1].

2. Background

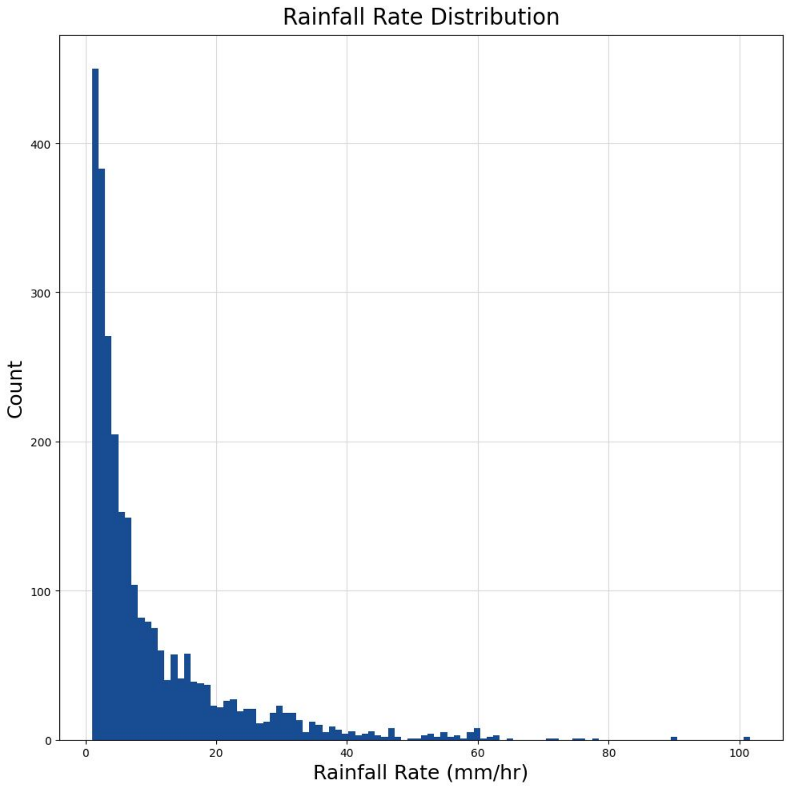
- **Z–R relationships** have been used since 1947 to estimate rainfall rate (R) using reflectivity (Z), and these equations vary slightly based on region and rain type [2].
    - $Z = 300R^{1.4}$ (WSR-88D Convective)
    - $Z = 200R^{1.6}$ (Marshall-Palmer)
- However, the commonly-used Z–R relationships fail to account for nuances in rainfall by precipitation type, region, and season [2].
- **Dual-polarization radar variables** better reflect the size, shape, and orientation of raindrops.
    - Using dual-polarization radar variables as input data, researchers have found that convolutional neural networks [5, 8] and random forest and regression tree methods [7] outperformed conventional Z–R relationships to estimate rainfall rate.
- In a study of deep-learning-based QPE models, rainfall estimates were more accurate when distinguishing rainfall intensity using a K_{DP} threshold [3].

3. Data

- Data from Central Oklahoma (June 8, 2022 and July 9, 2023) and South Florida (April 12, 2023) with significant rainfall:
  - **Dual-polarimetric radar data** from the Weather Surveillance Radar, 1988 Doppler (WSR-88D) at Level II.
  - **Rain gauge data** from the Oklahoma Mesonet and the South Florida Water Management District's DBHYDRO.



3. Data



Reflectivity Z (dBZ)

Differential reflectivity Z_{DR} (dB)

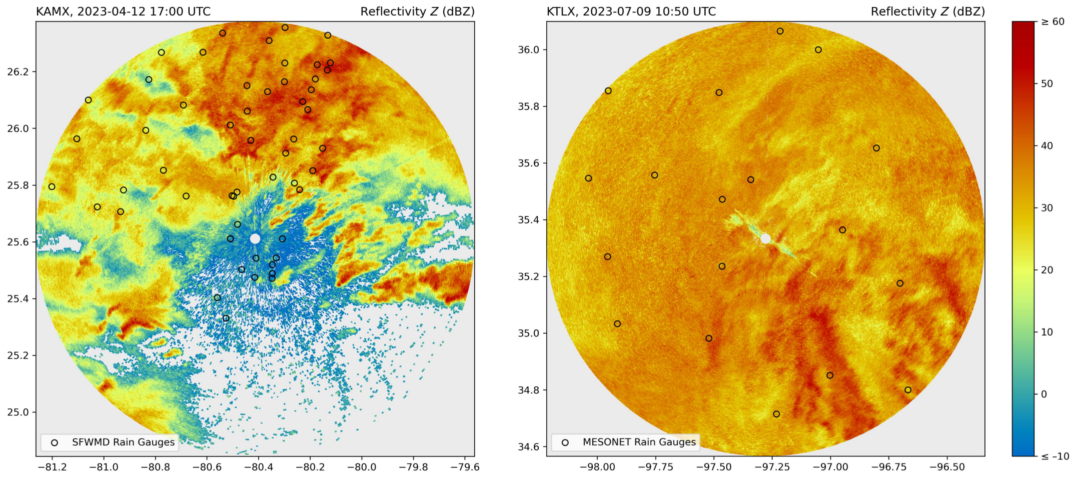
Specific differential phase K_{DP} (deg/km)

Co-polar correlation coefficient ρ_{hv}

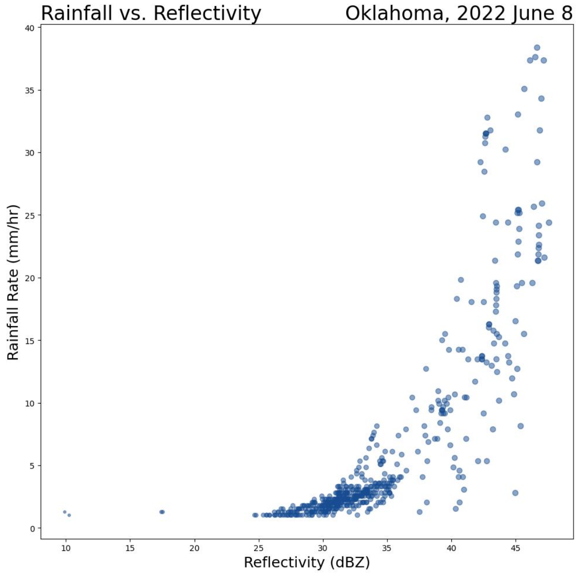
Rainfall rate (mm/hr)

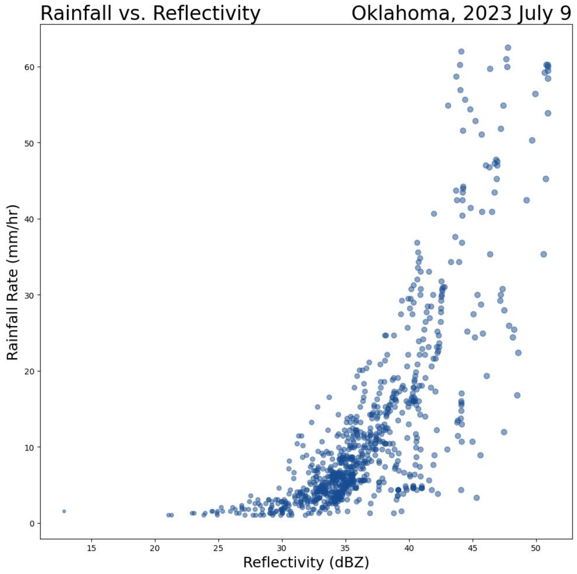
3.1 Reflectivity Z (dBZ)

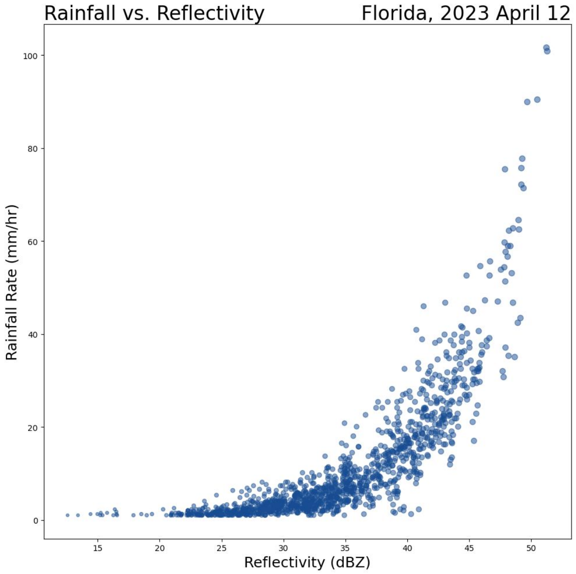
- Measures the amount of energy reflected back to the radar.
- Related to raindrop particle size and generally increases as rainfall rate increases [2].



3.1 Reflectivity–Rainfall (Z–R) Relationships

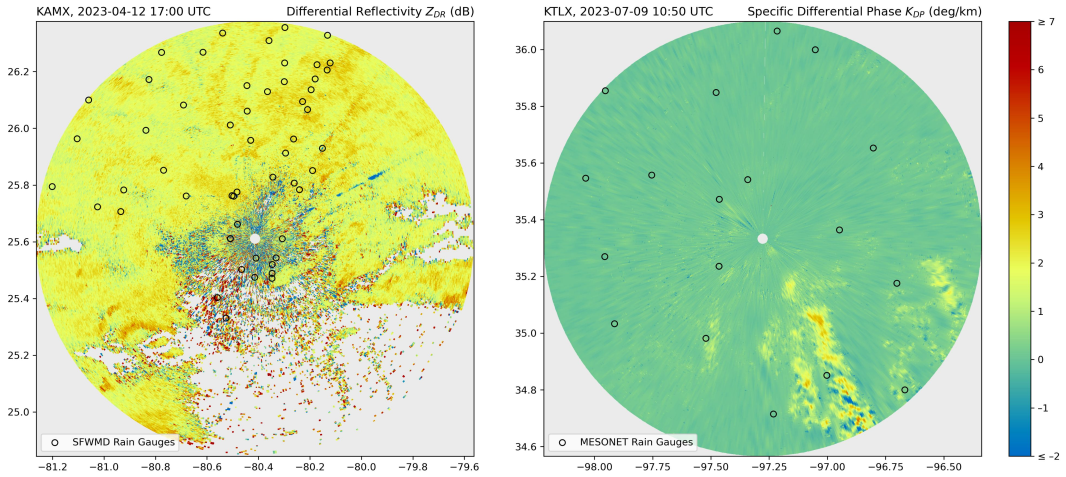






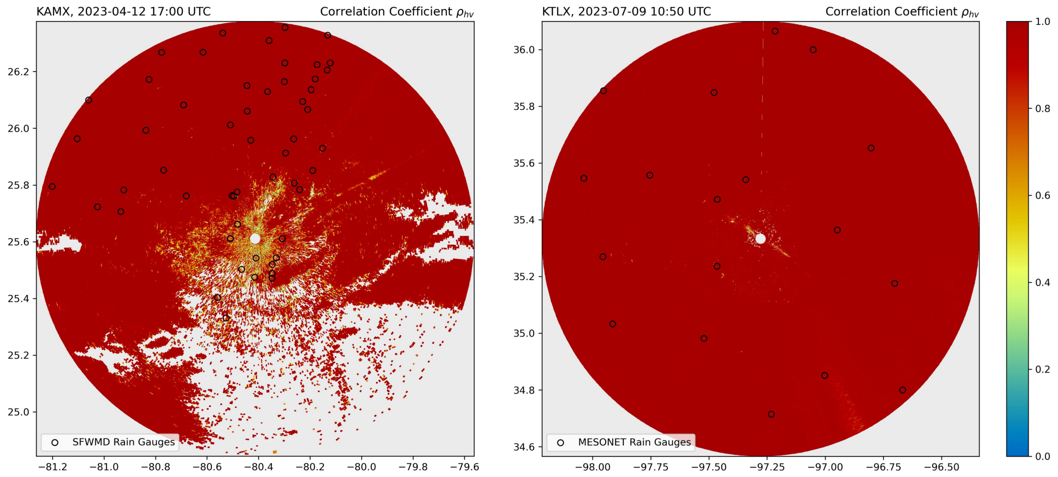
3.1 Differential reflectivity Z_{DR} (dB)

- Impacted by the composition or density of raindrops, helping differentiate water drops from ice pellets and snow [4].
- The ratio between reflectivity factors at horizontal and vertical polarizations.



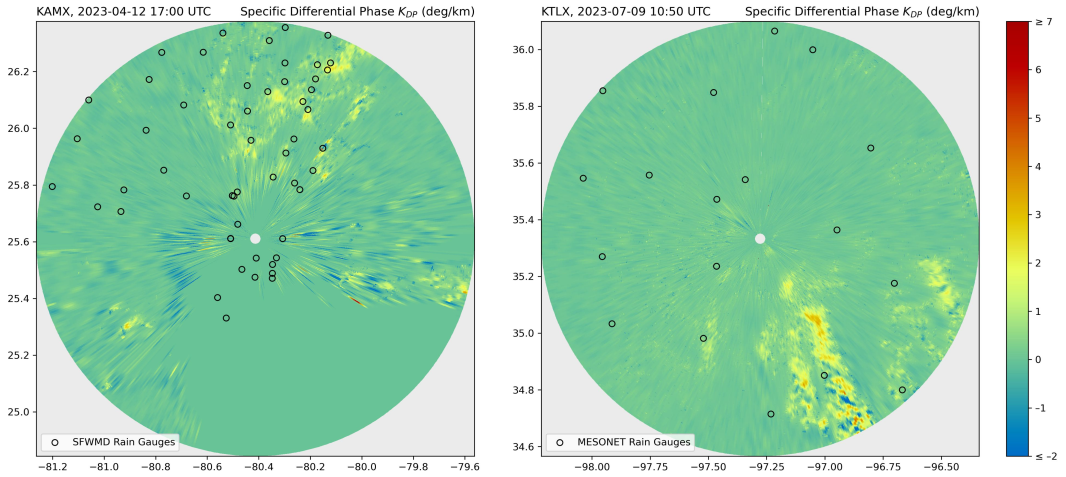
3.1 Co-polar correlation coefficient ρ_{hv}

- Measures variation in particle shapes and orientations [2, 4].
- Close to 1.0 during uniform rainfall and decreases with more variability in the types, shapes, and orientations of particles [4].



3.1 Specific differential phase K_{DP} (deg/km)

- Derived variable that represents the change in differential phase shift Φ_{DP} [2, 4].
- Useful for identifying heavy precipitation and when hail is mixed with rain, but can be noisy for light rain [4].



4. Methodology & Results

4.1 Benchmarking Symbolic Regression Algorithms

4.2 Symbolic Regression on Subsets of the Data Using Feyn

- Clusters (K-Means, Bisecting K-Means, Agglomerative Hierarchical)
- Decision Tree Leaf Nodes
- Grouping by Radar Variable Mean

4.3 Exploring New Symbolic Regression Models with gpg

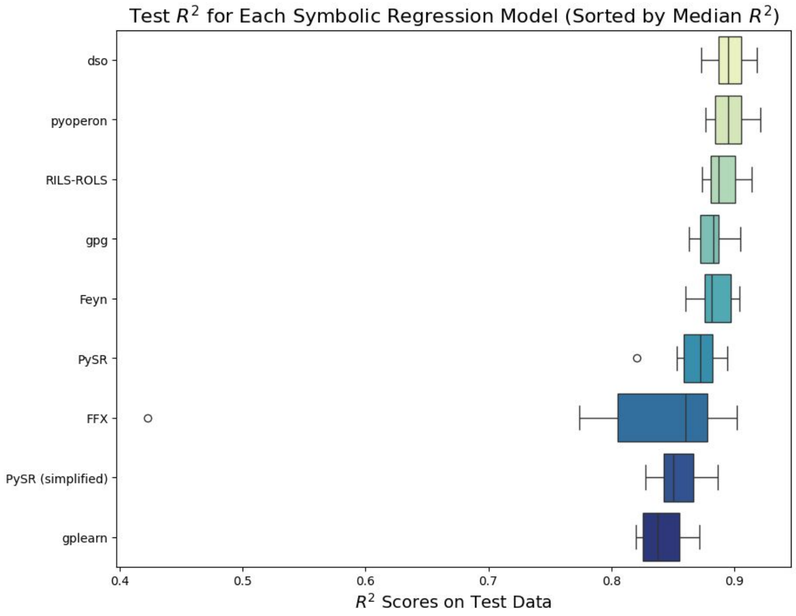
- Knowledge-based loss terms for gpg loss function

4.1 Benchmarking Symbolic Regression Methods

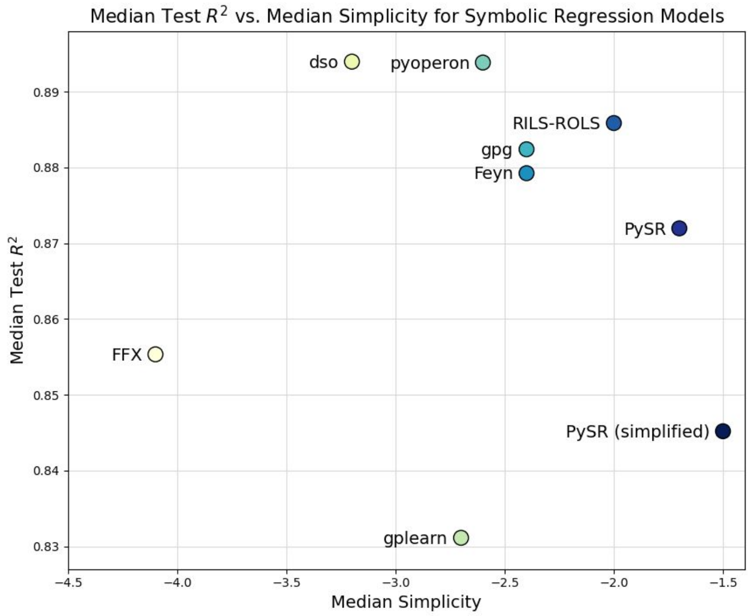
- Implemented eight symbolic regression methods based on criteria by La Cava et al. [1].
  - Genetic programming (gplearn, gpg, PySR, Feyn, pyoperon)
  - Deep learning (dso)
  - Other (FFX, RILS-ROLS)
- Run 10 trials with different training (75%) and testing (25%) sets.
- Analyzed accuracy using the coefficient of determination (R^2) and the normalized root square mean error (NRMSE).
- Analyzed equation complexity with a simplicity score indicating the number of components within the equation.

$$R^2 = 1 - \frac{\sum_{i=1}^{k}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{k}(y_i - \bar{y}_i)^2} \qquad NRMSE = \frac{\sqrt{\frac{1}{k}\sum_{i=1}^{k}(y_i - \hat{y}_i)^2}}{\bar{y}} \qquad simplicity = -log_5(s)$$

4.1 Benchmarking Symbolic Regression Methods



Each model's test R^2 scores over ten trials sorted by median test R^2

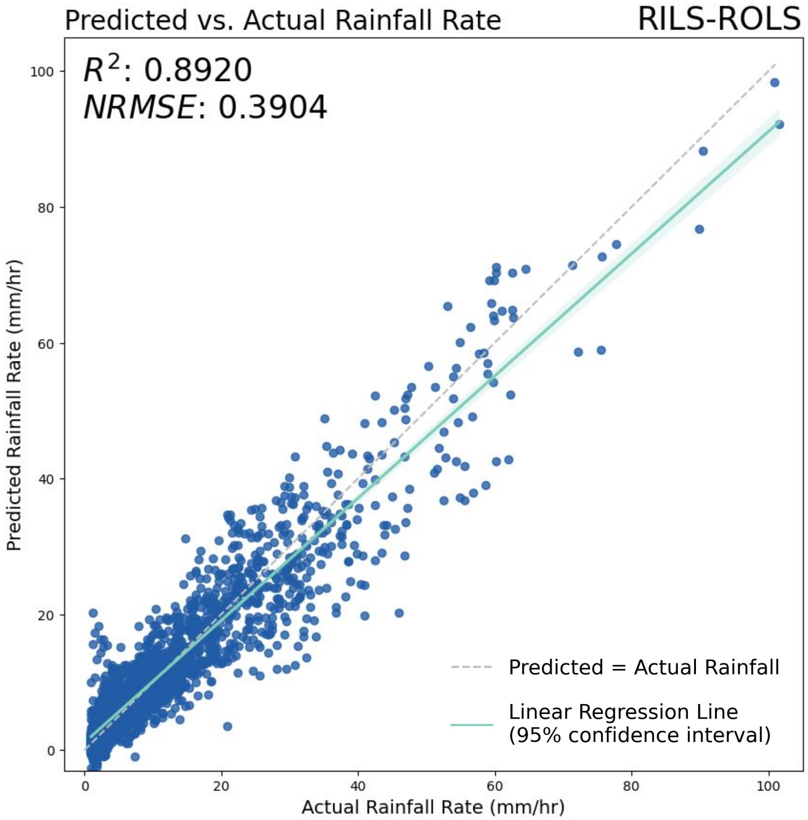


Each model's median test R^2 vs. median simplicity (simplicity closer to zero indicates simpler equations)

4.1 Benchmarking Symbolic Regression Methods

Equation with best combination of **accuracy and simplicity** using symbolic regression with RILS-ROLS

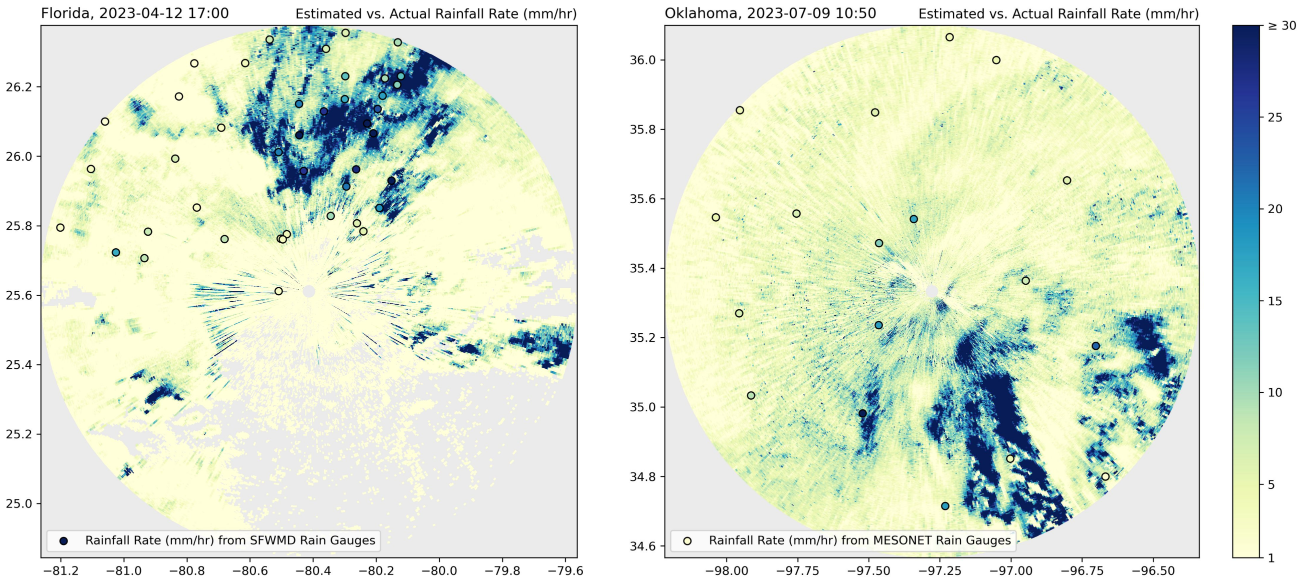
$$R = 1.208Z(K_{DP})\rho_{hv}^3 - 20.088K_{DP} + 2(10^{-6})\rho_{hv}^4 Z^4 e^{\cos(Z_{DR})} - 0.643$$



4.1 Benchmarking Symbolic Regression Methods

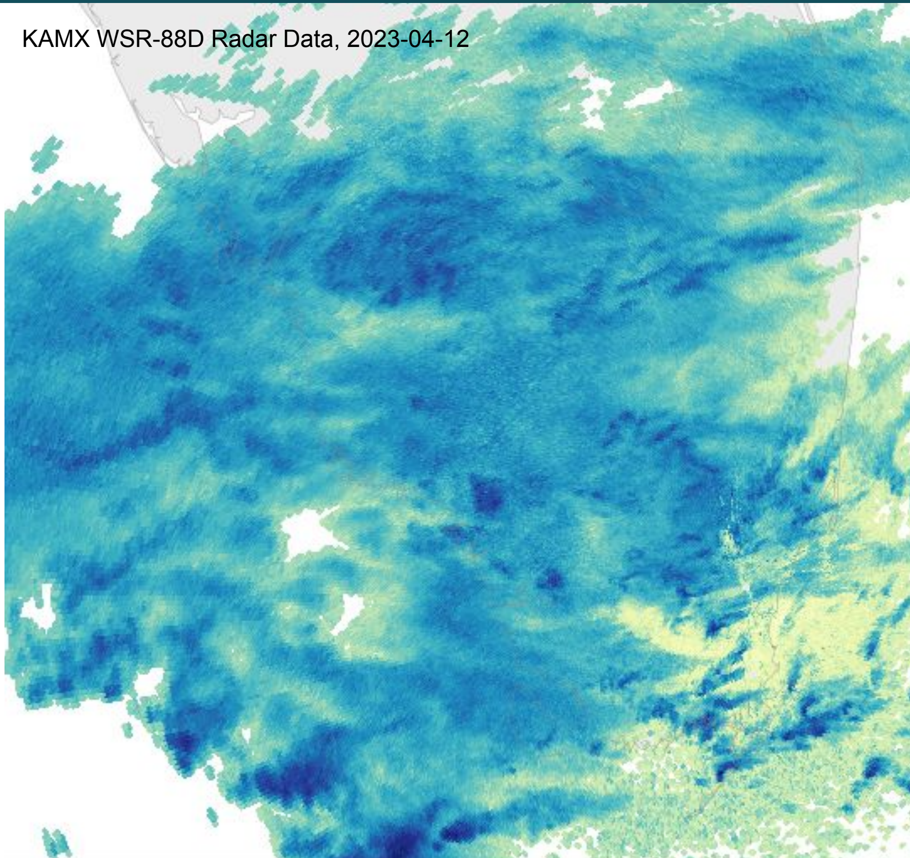
Predicted rainfall rate based on equation from RILS-ROLS

$$R = 1.208Z(K_{DP})\rho_{hv}^{3} - 20.088K_{DP} + 2(10^{-6})\rho_{hv}^{4}Z^{4}e^{\cos(Z_{DR})} - 0.643$$



4.2 Symbolic Regression on Subsets of Data

- One significant challenge to precipitation estimation is capturing **different precipitation types, distributions, and intensities**.
- Previous research has found that pre-processing the data to distinguish rainfall intensities has improved QPE accuracy [3].
- We test clustering algorithms, decision trees, and setting a threshold using Z_{DR} and ρhv to subset the data prior to running symbolic regression.

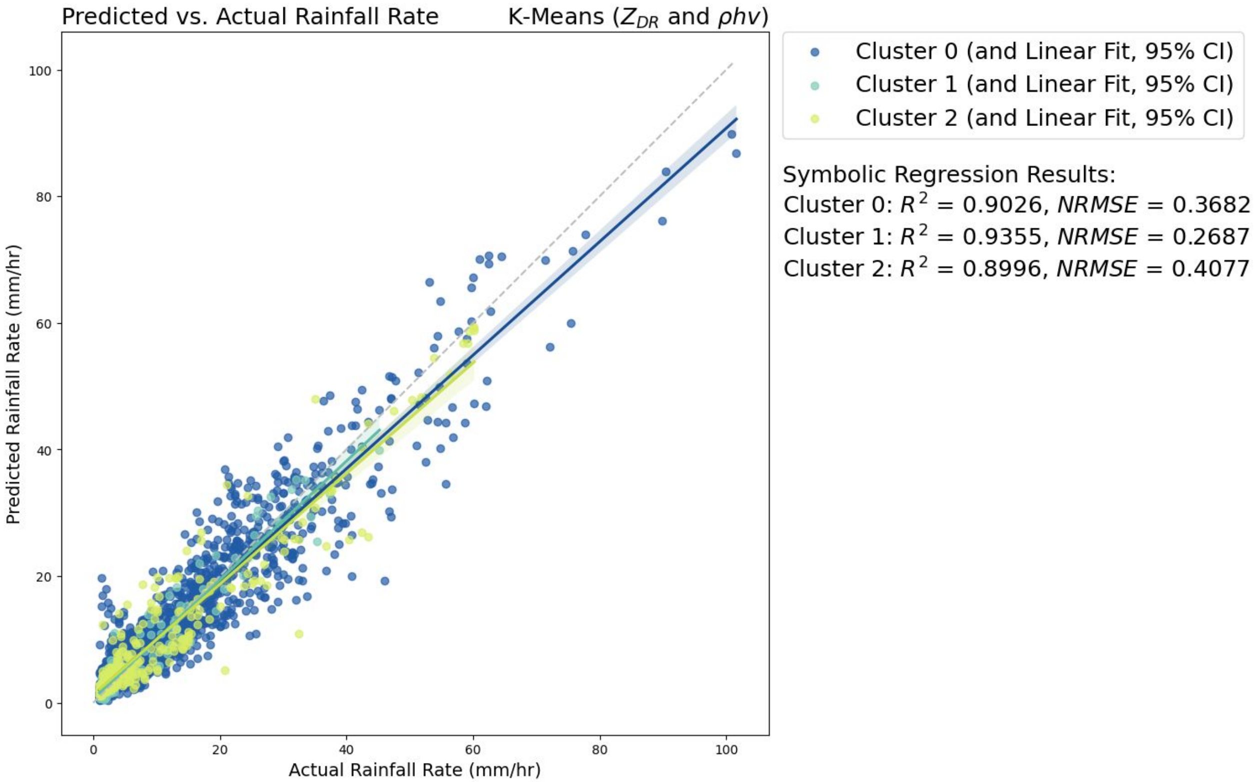


4.2 Symbolic Regression on Clusters

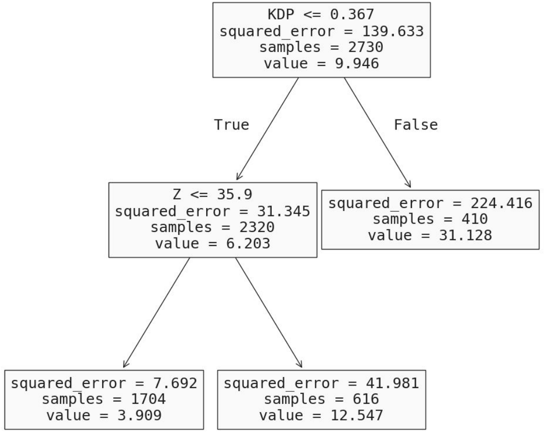
Mean metrics of three clusters from the trial with the highest mean test R^2 score for each clustering method using Feyn

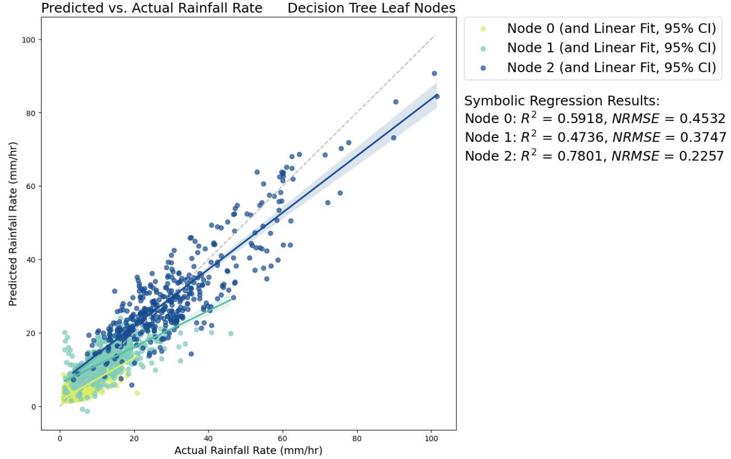
Cluster	Variable	Train R^2	Test R^2	Train NRMSE	Test NRMSE	Simplicity
All Data (Without Clustering)		0.8757	0.9046	0.4116	0.3817	-2.4
K-Means	All Radar	0.7382	0.7826	0.4142	0.3784	-2.1
	ρ_{hv} and Z_{DR}	0.9048	**0.9200**	0.3605	0.3250	-2.2
	Rain	0.6318	0.6764	0.2456	0.2434	-2.1
Bisecting K-Means	All Radar	0.7129	0.7527	0.4300	0.3936	**-2.0**
	ρ_{hv} and Z_{DR}	**0.9064**	0.8980	0.3554	0.3722	-2.1
	Rain	0.6106	0.6069	0.2214	0.2265	**-2.0**
Agglomerative	All Radar	0.7556	0.7887	0.4006	0.3727	-2.1
	ρ_{hv} and Z_{DR}	0.8973	0.8746	0.3758	0.3980	-2.3
	Rain	0.6466	0.6623	**0.2201**	**0.2248**	**-2.0**

4.2 Symbolic Regression on Clusters



4.2 Symbolic Regression on Decision Tree Leaf Nodes

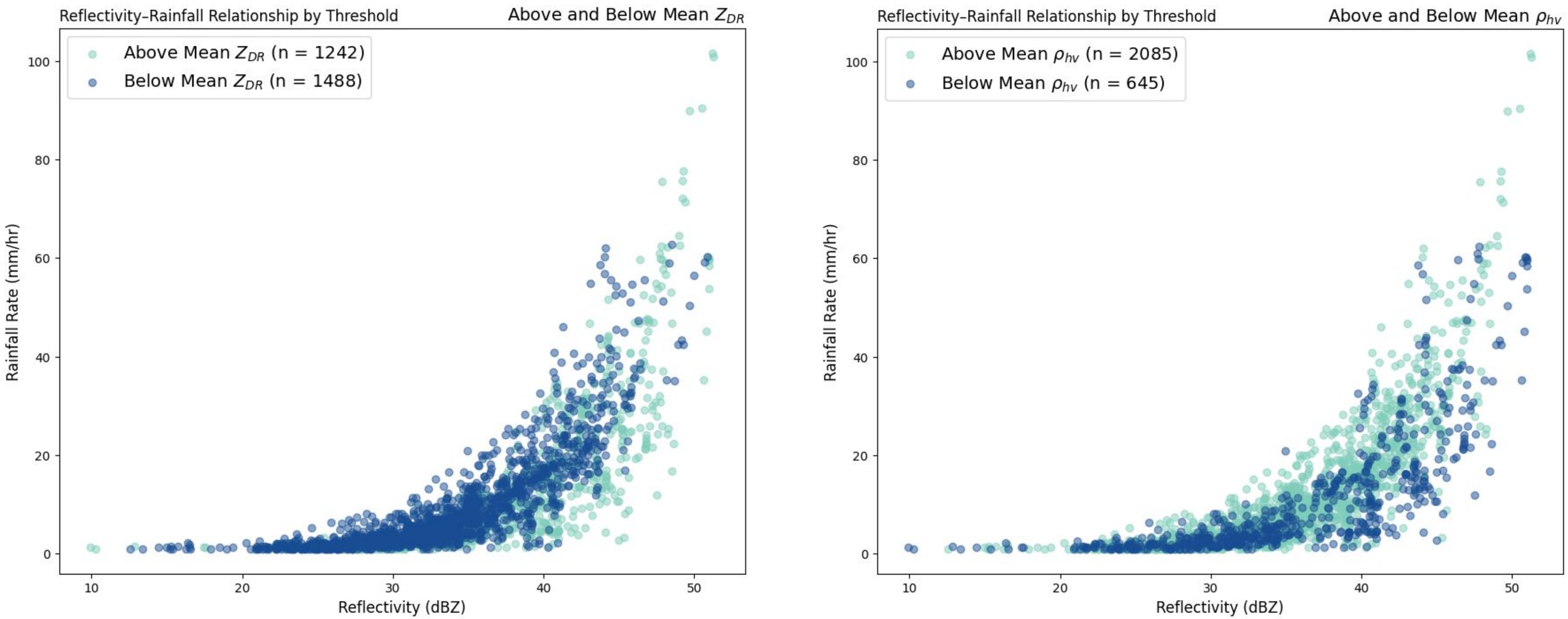




Metrics from the trial with the highest test R^2 score for each node using Feyn

Subset	Size	Train R^2	Test R^2	Train NRMSE	Test NRMSE	Simplicity
All Data	2730	0.8757	0.9046	0.4116	0.3817	-2.4
Node 1	1704	0.5775	0.6330	0.4620	0.4271	-2.1
Node 2	616	0.4510	0.5624	0.3923	0.3123	-2.1
Node 3	410	0.7773	0.7826	0.2277	0.2199	-2.0

4.2 Grouping by Radar Variable Mean

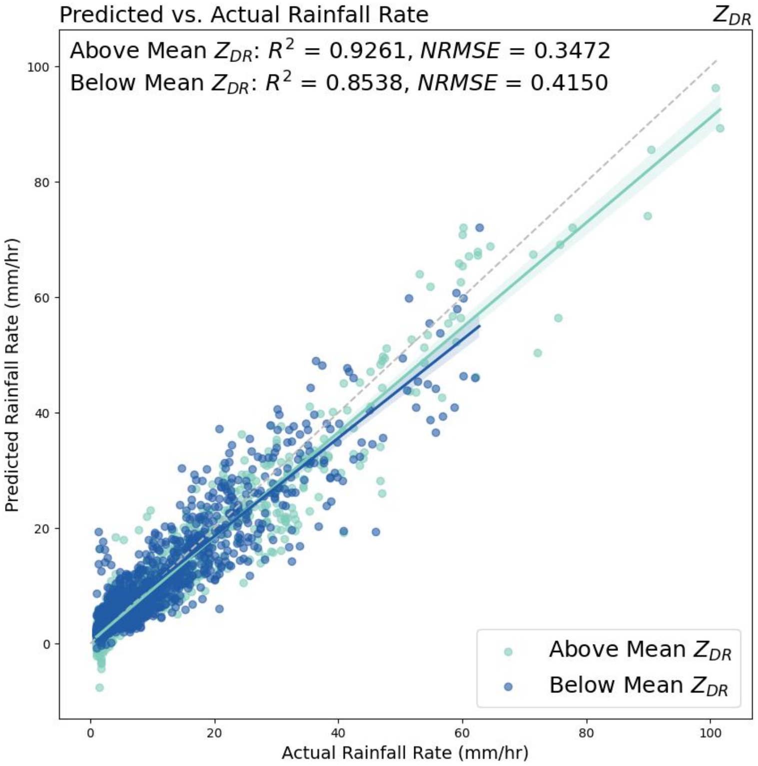


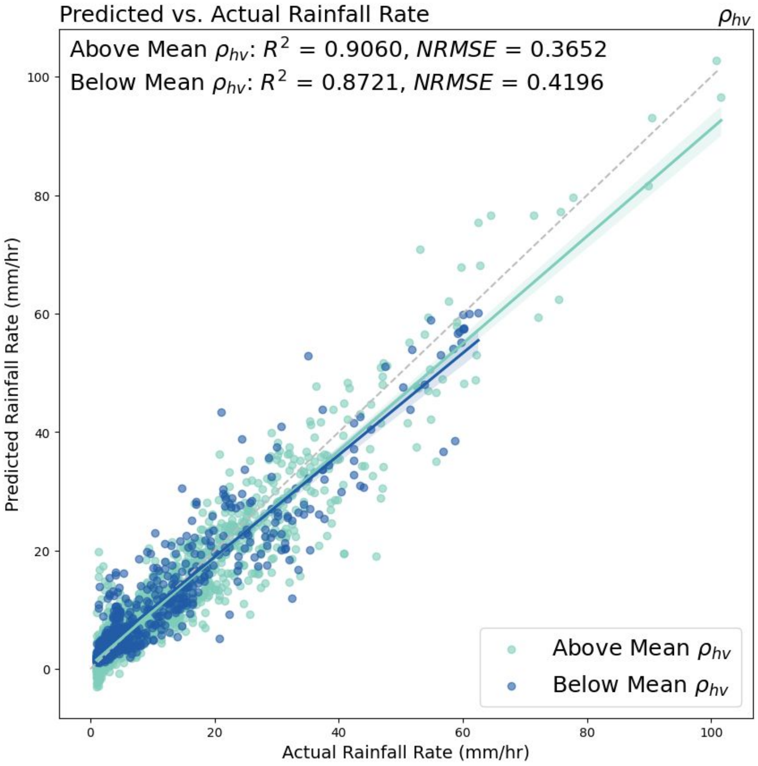
4.2 Grouping by Radar Variable Mean

Metrics from the trial with the highest test R^2 score for each group using Feyn

Variable	Group	Size	Train R^2	Test R^2	Train NRMSE	Test NRMSE	Simplicity
All Data		2730	0.8757	0.9046	0.4116	0.3817	-2.4
Z_{DR}	Above Mean	1242	**0.9161**	**0.9519**	0.3620	**0.2976**	-2.2
	Below Mean	1488	0.8538	0.8538	0.4096	0.4314	-2.3
ρ_{hv}	Above Mean	2085	0.9023	0.9132	**0.3554**	0.3898	-2.1
	Below Mean	645	0.8586	0.9025	0.4268	0.3985	-2.4

4.2 Grouping by Radar Variable Mean





4.3 Exploring New Symbolic Regression Models with gpg

- Incorporated knowledge-based loss terms [6] into the loss function of gpg
    - Z−R relation ($Z = aR^b$) loss term: $loss_f = loss(Y, \hat{Y}) + \lambda * loss(\hat{Y}, (\frac{Z}{a})^{\frac{1}{b}})$
    - Cluster-based loss term: $loss_f = loss(Y, \hat{Y}) - \lambda * silhouette\_score(\hat{Y}, \mathrm{L})$
        - Silhouette score: measures how well the rainfall rates are assigned to their predetermined clusters
    - Binned rainfall loss term: $loss_f = loss(Y, \hat{Y}) + \lambda * [loss(\widehat{Y_j}, L_j) + loss(\widehat{Y_j}, U_j)]$
        - Prior to training: split the data into three groups of low, medium, and high rainfall rate
        - Adds to the loss if the model predicts a rainfall rate not aligning with the groups `

Y = ground-truth rainfall rate
$\hat{Y}$ = predicted rainfall rate
Z = Reflectivity (mm^6/mm^{-3})
L = cluster labels
$\widehat{Y_j}$ = predicted rainfall rate for group $j = 1, 2, 3$
L_j = lower bound for group $j = 1, 2, 3$
U_j = upper bound for group $j = 1, 2, 3$
λ = weight parameter

4.3 Exploring New Symbolic Regression Models with gpg

Metrics from the model with the highest test R^2 score using custom loss functions in gpg

Loss Function	Train R^2	Test R^2	Train NRMSE	Test NRMSE	Simplicity
Original	0.8744	0.9049	0.4115	0.3842	-2.5
Z-R ($\lambda = 1$) (Equation 4.3.1)	0.8546	0.8900	0.4427	0.4132	**-2.3**
Silhouette score ($\lambda = 20$) (Equation 4.3.2)	0.8746	0.9060	0.4110	0.3819	-2.5
Binned rainfall ($\lambda = 0.01$) (Equation 4.3.3)	**0.8748**	**0.9067**	**0.4108**	**0.3804**	**-2.3**

- Including the binned rainfall term in the loss function generated a more accurate and less complex symbolic expression

5. Conclusion

Benchmarking

- Symbolic regression is effective for quantitative precipitation estimation, providing interpretable and accurate models.

Symbolic Regression on Subsets with Feyn

- There is potential for data pre-processing methods that subset the data to improve the accuracy of learned equations.
- Applying Feyn symbolic regression on three clusters resulting from k-means clustering based on Z_{DR} and ρ_{hv} achieved improved R^2 scores, lower NRMSE scores, and slightly simpler equations.

Custom Loss Functions with gpg

- Adjusting and applying custom loss functions in gpg slightly improved the R^2 scores and NRMSE scores while also improving the model simplicity.

5.1 Further Work

This study can be built upon in the following ways:

- Test symbolic regression models on a larger dataset encompassing more geographic regions and dates.
- Explore symbolic regression on time series data.
- Conduct a deeper analysis on how to incorporate domain knowledge into the loss function of symbolic regression models to further improve learned equations.

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Thank You!